

CIRCULAR RESONATOR

These relations pertain to the modes of third sound in a film, thickness h , adsorbed on the top and bottom surfaces of a disk, radius a . Only modes symmetric with respect to the top and bottom are considered.

Bessel Functions

The surface displacements η are Bessel functions of the first kind...

$$\eta(r, \phi) = J_m(kr) \cdot \cos(m\phi) \quad \text{with} \quad \frac{d}{dx} J_m(x) = 0 \quad \text{at} \quad x_{mn} \quad k = \frac{x_{mn}}{a}$$

Here's a good guess for the roots... $m := 1 \quad n := 1 \quad x := \pi \cdot \left(n + .674 \cdot m^{.844} - .178 \cdot \frac{m^{.768}}{n} - .956 \right)$

clean the root up $x_{mn} := \text{root}\left(\frac{d}{dx} J_n(m, x), x\right) \quad x_{mn} = 1.84118$

value at the root $J_{mn} := J_n(m, x_{mn}) \quad J_{mn} = 0.58187$

RMS displacement over area $J_{rms} := \sqrt{\frac{1}{2} \cdot \left[1 - \left(\frac{m}{x_{mn}} \right)^2 \right]} \cdot J_{mn}^2 \quad J_{rms} = 0.34547$

Resonator and Film (MKS units)

film thickness $h_0 := 3 \cdot 10^{-9}$ third sound speed $c := 10$ He density $\rho := 145$

mode amplitude $\eta := 1 \cdot 10^{-10}$ disk radius $a := .00615$

$k := \frac{x_{mn}}{a} \quad \omega := c \cdot k \quad k = 299.37941 \quad \frac{\omega}{2 \cdot \pi} = 476.47714$

Energies... $E_{vdw} = \int_{\text{cell}} \frac{1}{2} \cdot \rho \cdot h^2 \cdot dA \quad E_{kin} = \int_{\text{cell}} \frac{1}{2} \cdot \rho \cdot h \cdot v^2 \cdot dA$

Standing Waves

film surface $h(r, \phi, t) := h_0 + \eta \cdot J_n(m, k \cdot r) \cdot \cos(m \cdot \phi) \cdot \cos(\omega \cdot t)$

flow velocity $v(r, \phi, t) := c \cdot \frac{\eta}{h_0} \cdot \left[\begin{array}{l} -\frac{1}{k} \frac{d}{dr} J_n(m, k \cdot r) \cdot \cos(m \cdot \phi) \\ \frac{m}{k \cdot r} \cdot J_n(m, k \cdot r) \cdot \sin(m \cdot \phi) \end{array} \right] \cdot \sin(\omega \cdot t)$ peak speed in azimuthal (lower) component

lateral element displacements $\delta(r, \phi, t) := \frac{\eta}{k \cdot h_0} \cdot \left[\begin{array}{l} -\frac{1}{k} \frac{d}{dr} J_n(m, k \cdot r) \cdot \cos(m \cdot \phi) \\ \frac{m}{k \cdot r} \cdot J_n(m, k \cdot r) \cdot \sin(m \cdot \phi) \end{array} \right] \cdot \cos(\omega \cdot t)$

wave energy (each surface) $E = E_{\text{vdw}} \cdot \cos(\omega \cdot t)^2 + E_{\text{kin}} \cdot \sin(\omega \cdot t)^2$ $E := \frac{1}{2} \cdot \rho \cdot c^2 \cdot \pi \cdot a^2 \cdot h_0 \cdot \left(\frac{\eta}{h_0}\right)^2 \cdot J_{\text{rms}}^2$

$E = 3.42711 \cdot 10^{-13}$ no angular momentum

Travelling Waves

film surface $h(r, \phi, t) := h_0 + \eta \cdot J_n(m, k \cdot r) \cdot \cos(m \cdot \phi - \omega \cdot t)$

flow velocity $v(r, \phi, t) := c \cdot \frac{\eta}{h_0} \cdot \left[\begin{array}{l} \frac{1}{k} \frac{d}{dr} J_n(m, k \cdot r) \cdot \sin(m \cdot \phi - \omega \cdot t) \\ \frac{m}{k \cdot r} \cdot J_n(m, k \cdot r) \cdot \cos(m \cdot \phi - \omega \cdot t) \end{array} \right]$ peak speed in azimuthal (lower) component

lateral element displacements $\delta(r, \phi, t) := \frac{\eta}{k \cdot h_0} \cdot \left[\begin{array}{l} \frac{1}{k} \frac{d}{dr} J_n(m, k \cdot r) \cdot \cos(m \cdot \phi - \omega \cdot t) \\ -\frac{m}{k \cdot r} \cdot J_n(m, k \cdot r) \cdot \sin(m \cdot \phi - \omega \cdot t) \end{array} \right]$

wave energy (each surface) $E = E_{\text{vdw}} + E_{\text{kin}}$ $E := 2 \cdot \frac{1}{2} \cdot \rho \cdot c^2 \cdot \pi \cdot a^2 \cdot h_0 \cdot \left(\frac{\eta}{h_0}\right)^2 \cdot J_{\text{rms}}^2$

$E = 6.85421 \cdot 10^{-13}$

wave angular momentum

$$L = \int_{\text{cell}} (\mathbf{r} \times \mathbf{v}) \cdot \rho \cdot (h_0 + \eta) dA = \frac{m}{\omega} \cdot \rho \cdot c^2 \cdot \pi \cdot a^2 \cdot h_0 \cdot \left(\frac{\eta}{h_0}\right)^2 \cdot J_{\text{rms}}^2 = \frac{m}{\omega} \cdot E$$

$L := \frac{m}{\omega} \cdot E$ $L = 2.28947 \cdot 10^{-16}$

quantization properties (each surface) $h_b := 1.054 \cdot 10^{-34}$ $m_4 := 6.646 \cdot 10^{-27}$ $\frac{\eta}{h_0} = 0.03333$

harmonic oscillator excitation level $n := \frac{E}{h_b \cdot \omega}$ $n = 2.17218 \cdot 10^{18}$

number of particles $N := \frac{\rho \cdot \pi \cdot a^2 \cdot h_0}{m_4}$ $N = 7.7773 \cdot 10^{15}$