SIMPLE HARMONIC OSCILLATIONS $\left(e^{i\cdot\omega\cdot t}\right)$

The third sound resonances are observed by two methods. Slowly scanning the drive frequency at a fixed voltage (CW) and monitoring the response results in data for complex amplitude vs. frequency. Driving near resonance, turning off the drive and detecting the decaying response is a free decay.

The CW reponse is a standard driven, damped response...

resonant frequency $f_0 := 440$

resonant amplitude $A := 1$

quality factor $Q := 10000$

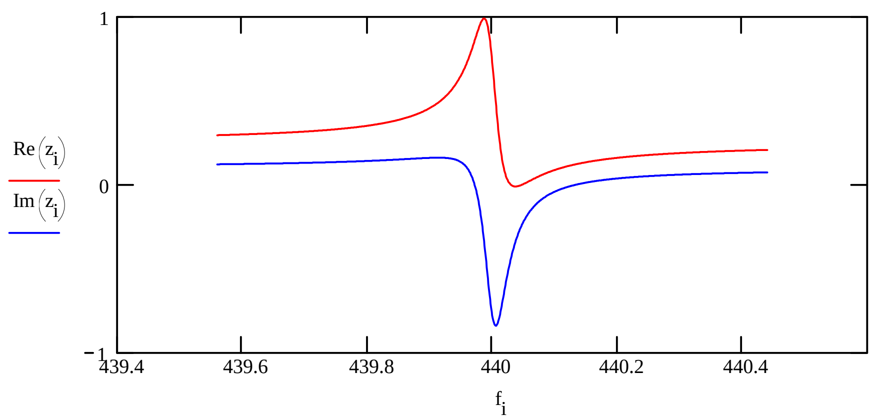
with some arbitrary electronic phase shift and background pick-up due to other coupling to the drive. $\phi := .5 \quad x_0 := .25 \quad y_0 := .1$

$$x(f) := \frac{A\cdot\frac{f_0}{f}\cdot\exp(i\cdot\phi)}{Q\cdot\left(\frac{f_0}{f}-\frac{f}{f_0}\right)+i}+x_0+i\cdot y_0$$

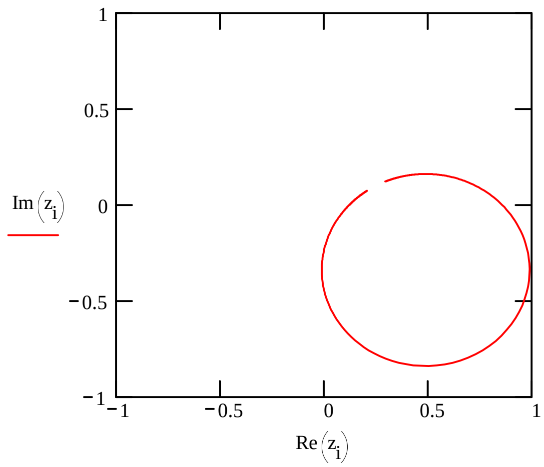
Take a look at a plot...

$$npts := 1000 \qquad fspan := \frac{f_0}{Q}\cdot 20 \qquad df := \frac{fspan}{npts}$$

$$i := 0..npts \qquad f_i := f_0 - .5\cdot fspan + \frac{i}{npts}\cdot fspan \qquad z_i := x(f_i)$$



The imaginary vs. real plot is a circle...



CW data can be fit to the above functions to extract f_0 and Q.

A free decay is detected relative to some reference frequency so the observed signal oscillates at the difference frequency. There is no background since the drive is off, and the initial phase shift is ignored.

$$f_r := 441 \qquad \Delta\omega := 2\cdot\pi\cdot\left(f_0 - f_r\right) \qquad \omega_0 := 2\cdot\pi\cdot f_0$$

$$s(t) := A\cdot\exp(i\cdot\Delta\omega\cdot t)\cdot\exp\left(\frac{-\omega_0\cdot t}{2\cdot Q}\right)$$

The phase drift of the detected signal gives the resonant frequency (relative to the reference) and a log vs. time plot of the amplitude gives the Q.

CW data is useful when the free decay is short compared to the time needed to observe the signal within the experimental noise level. Free decays are more convenient when noise is not a problem and quick checks of the frequency and Q are desired.