Model for Cyclically Astable Third Sound Resonance

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Abstract. Third sound in a circular resonator is capable of rich behavior due to the coincidence of several characteristics: a high quality factor; the capability for wave-induced circulation changes; vortex pinning on the substrate; and the Doppler shifting of modes. One dimensional models for each of these components have previously been used to reproduce highly nonlinear third sound CW lineshapes using a steady-state approximation. We now include oscillator transients to account for cyclical amplitude modulations of third sound recently observed under conditions of a drive force where both the amplitude and frequency are fixed.

Keywords: superfluidity, circulation, vortices, pinning, third sound
PACS: 67.40 Hf, 67.40 Vs

INTRODUCTION

Third sound is a unique tool for the study of vortices in superfluid helium films. Large amplitude wave agitation can be used to create and destroy circulatory persistent current states in a resonator and the Doppler effect can be used to determine the corresponding net vorticity.[1] The persistence of flow concurrent with intermingled vortices demonstrates a remarkably strong pinning of the vortices to the substrate.

The high Q typical of third sound at low temperatures allows small changes in the resonant frequency to have dramatic consequences. There are two dominant sources of amplitude dependent frequency shifts that contribute to distorted resonance lineshapes. First, there is a quadratic mode coupling interaction associated with the AC Bernoulli pressure[2], and second, high amplitude wave agitation can induce changes in any persistent current present in the resonator. Both of these effects are illustrated in Figure 1.

Consider first a scan up through a resonance. The amplitude rises as resonance is approached, but the mode coupling effect shifts the frequency down. At a high enough drive amplitude, this results in a catastrophic jump of mode to an above-resonant situation. The scan then proceeds along the above-resonant tail to a lower amplitude. The resonance peak is completely bypassed. A scan down is qualitatively different. The mode coupling shift moves the resonance further down and elongates the eventual reaching of a peak. Just past the peak, the steady-state condition is unstable. The amplitude drops concurrent with the resonant frequency shifting back up, and the scan continues from a point on the below-resonant tail.

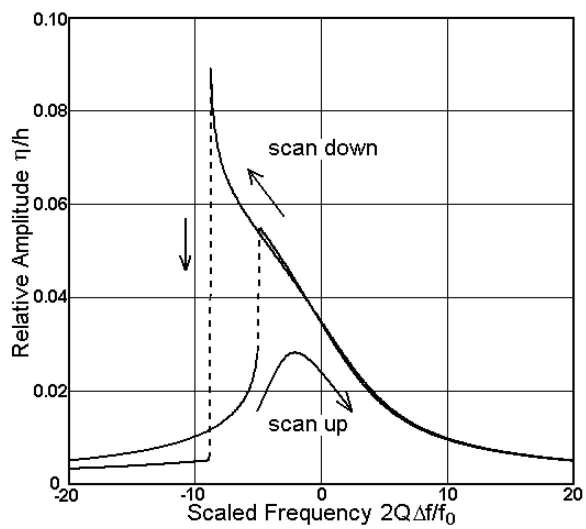

FIGURE 1. Distortion and catastrophic jumps are a consequence of amplitude dependent frequency shifts. The drive frequency is Δf relative to the linear resonance at f_0.

Induced circulation adds another source of distortion to the lineshape. As the amplitude increases beyond a threshold for de-pinning, the wave agitation first diminishes the circulation. At higher amplitudes, new vorticity is induced and the circulation increases.

CP850, *Low Temperature Physics: 24th International Conference on Low Temperature Physics;*
edited by Y. Takano, S. P. Hershfield, S. O. Hill, P. J. Hirschfeld, and A. M. Goldman

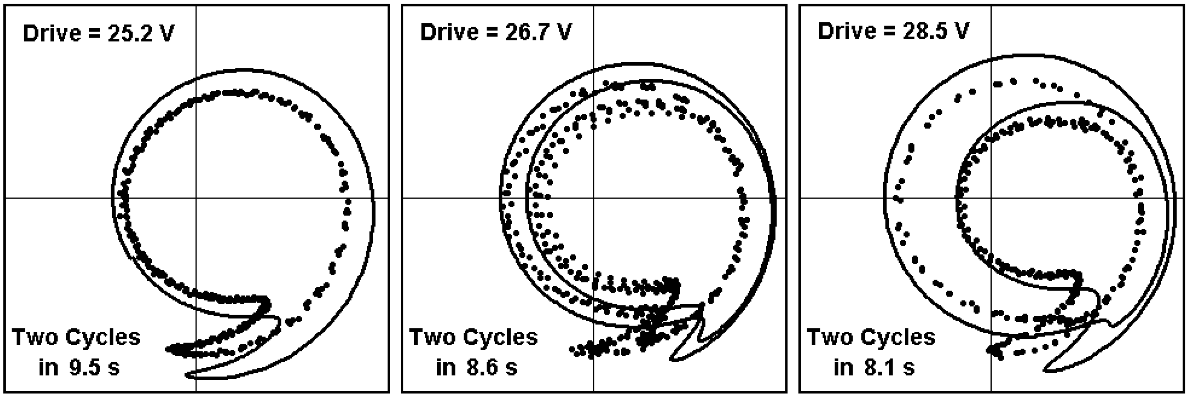

FIGURE 2. Real and imaginary parts of the cyclical data and model at different drives. Full scale is $\eta/h=0.15$ in all plots.

The sharp peak of the down scan results as the increasing circulation moves the mode faster into resonance. The jump to the below-resonant tail leaves the mode permanently shifted up to about $Q\Delta f/f_0=11$.

An interesting situation arises which has been experimentally observed[3]. With a fixed drive, it is possible to have a temporally unstable response. With the resonance above the drive, moderate wave agitation causes the circulation to decay. This shifts the mode through resonance, where the higher wave amplitude causes the circulation to increase. The mode is consequently shifted back up above the drive, initiating a new cycle. This repetition is the basis for the cyclically astable third sound resonance.

MODEL AND RESULTS

A steady state model for the interplay between resonance and circulation has been successfully applied to reproduce highly distorted lineshapes[4]. It captures the physics of the circular resonator and its spatially dependent circulation with two parameters, the wave-amplitude to film-thickness ratio, $z=\eta/h$, and a radially averaged circulation flow, scaled to the third sound speed, $\chi=\langle v_{circ}\rangle/c_3$. Flow changes are the result of wave-flow coupling dependent on both χ and z. The amplitude z is then self-consistently determined from the steady state response shifted by the mode coupling (amplitude z) and Doppler effects (flow χ).

This model is now extended to include the transient behavior that dominates the cyclically astable resonances. The amplitude z is taken to be complex and a discretized stepping of the simple harmonic oscillator with transients is performed, updating the oscillator parameters (resonance frequency and Q) appropriate to the flow χ and amplitude magnitude $|z|$. The transient aspect of the model is approximate only to the extent that the parameters change during the time steps.

Figure 2 shows experimental data together with the model behavior for a 12.3 mm dia. resonator with a gold substrate. Shown are the real and imaginary parts of the complex amplitude driven at 1670 Hz with c_3=21 m/s at 0.6 K. The paths cycle about the origin with the times for two revolutions shown. Three drive amplitudes are shown with all other conditions fixed. The flow was approximately χc_3=11 cm/s.

The model allows us to verify exactly what is happening during the cycling. Most of the cycle is spent with the third sound mode frequency above the drive. These are the major circular sections in the figures, dominated by a free oscillator transient and a slowly decaying circulation. The hook near the negative imaginary axis corresponds to where the mode moves down through the drive. The amplitude builds up and the circulation then increases moving the mode back above resonance. No longer driven near resonance, the amplitude decays and the circulation reverts to a slow decay. The model shows a flow variation of about 20% over the course of the cycling. The model also indicates that vortex de-pinning occurs with an exponential activation[4] of the form $\exp(bv_{flow})$ with b=12±5 s/m.

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