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Vortex Equilibrium in Film Flow Near a Defect

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A simple model based on numerical simulations in bulk is used to examine the pinning characteristics of vortices in helium films adsorbed in the vicinity of substrate defects. The film profile in the absence of a vortex is first calculated including both the Van der Waals potential of a bump defect and surface tension. The displacement and ultimate de-pinning of a vortex line constrained within the profile is then followed as a function of an imposed superfluid flow. The behavior of the vortices lends insight into recent experimental results sensitive to the presence of pinned vortices and indicates that the relevant pinning sites are atomic in dimension.

1. INTRODUCTION

Vortex pinning in films has long been implicated as an influencing feature in the dynamics of superfluid film dissipation.[1] More recently, pinned vortices have been shown to be quite stable at low temperatures, even in the presence of large background film flow conditions.[2] This stability naturally leads one to ask if it is reasonable that actual substrate features can be responsible for the observed pinning. We present results from a simple model to estimate the vortex pinning characteristics on arbitrary one dimensional substrate features.

Two substrate bump shapes will be discussed in this paper: a Gaussian and a circular arc profile with varying widths and heights relative to the film thickness. The bumps are taken to be one dimensional, making them long ridges with the profiles taken as perpendicular cuts through the ridge. The externally imposed flow is assumed to be independent of position and running parallel to the ridges. Equilibrium configurations of a vortex will be found as a function of the magnitude of the external flow. We find both the critical velocity above which no equilibrium exists and a "precession frequency" related to the vortex response to low flow speeds.

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In bulk, more complete studies of vortex behavior have motivated some of the simplifications that we will make.[3] In the presence of a flow, a vortex line will move with the flow. The flow consists of two parts, an externally imposed component along with a component induced by the vortex itself. This self induced flow depends on the curvature of the vortex core. A vortex pinned on a substrate bump in equilibrium with an external flow must be shaped so that the external and self induced flows exactly cancel. The self induced flows must include virtual lines, or image lines associated with boundaries. This dictates that the vortices intersect any boundary normally.

The free surface of a film makes the vortex problem in the film more difficult. Substrate defects will modify the Van der Waals potential in their vicinity and in turn, change the film profile. In addition, the presence of the vortex's own flowfield will modify the film profile, creating some form of dimple over a length scale of roughly the film thickness.

In our numerical calculation, we treat the fluid classically. We first find the equilibrium film surface profile by balancing the Van der Waals potential against surface tension. This surface shape remains fixed for the rest of the process, effectively ignoring any contribution that the vortex dimple may have. The vortex is then taken as a section of a ring whose radius is such that the ring would propagate through bulk fluid at the imposed flow speed, given a singly quantized circulation strength. The position within the film where the ring intersects both the free surface and the substrate perpendicularly is the assumed equilibrium position.

This is a somewhat crude model for the messy hydrodynamics that would be required to solve the complete classical problem, but its simplicity allows estimates of pinning properties on a variety of substrate shapes with relatively little computational effort.

2. NUMERICAL DETAILS

The Van der Waals potential per unit volume of helium liquid is found by integrating an r^{-6} potential over the substrate, and is itself a function of position:

$$U(\bar{r}) = -\frac{6\alpha}{\pi}\int \frac{dV'}{|\bar{r}' - \bar{r}|^6} \tag{1}$$

Here α is defined so that $-\alpha/y^3$ is the potential a distance above a perfectly flat substrate. For a one dimensional substrate described by a height profile f(x'), the integral in Eq. (1) reduces to a one dimensional integral over x' as an additive term to the explicit flat plane contribution.

$$U(x,y) = -\frac{\alpha}{y^3} - \frac{3\alpha}{4}\int_{-\infty}^{\infty} \frac{dx'}{(x-x')^4}\left[u\frac{3(x-x')^2 + 2u^2}{\left((x-x')^2 + u^2\right)^{\frac{3}{2}}} \right]_{u=y-f(x')}^{u=y} \tag{2}$$

This above integral is numerically evaluated, taking care through the region x~x', to give the potential function for the field above the bump.

If there were no surface tension, the film profile would be determined by the equipotential surfaces of the function U(x,y). With surface tension, the profile is determined by satisfying Laplace's formula involving the surface tension γ:

$$-\frac{\gamma}{R}+U(x,y)=-\frac{\alpha}{h_0^3} \qquad \text{with the curvature} \quad \frac{1}{R}=\frac{y''}{\left(1+y'^2\right)^{3/2}} \quad . \tag{3}$$

This differential equation is numerically solved subject to the boundary condition that the solution becomes asymptotic to $y(x)=h_0$, the flat film thickness, far from the bump. Figure 1 shows the film profile for a circular bump at several film thicknesses.

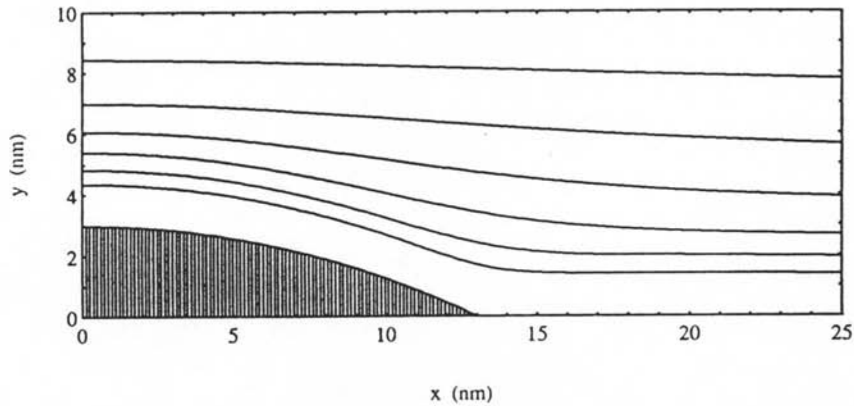

Fig. 1 Film profiles over a circular bump for flat films of 1.4, 2.0, 2.7, 3.7, 5.1, and 7.8nm. Both the Van der Waals potential and surface tension are included.

Once the film profile over the bump is determined, if it is assumed that it is not modified by the vortex, then the same profile can be used to find the vortex position vs. film flow speed. A point on the free surface is chosen and a family of circular arcs normal to the free surface through that point is considered. In general, only one of these arcs will penetrate the substrate perpendicularly. This is taken as the equilibrium vortex position in a flow speed given by the speed of an equivalently curved singly quantized vortex ring,

$$v(R)=\frac{K}{4\pi R}\left[\ln\left(\frac{8R}{a_0}\right)-\frac{1}{4}\right] \tag{4}$$

and the vortex displacement is recorded as the average of the lateral intersection points with the free surface and substrate. The quantum of circulation is K and a_0 is the core radius of .13nm. Each choice of initial point on the free surface leads to a flow velocity and displacement, and these values are plotted. An example is shown in Fig. 2.

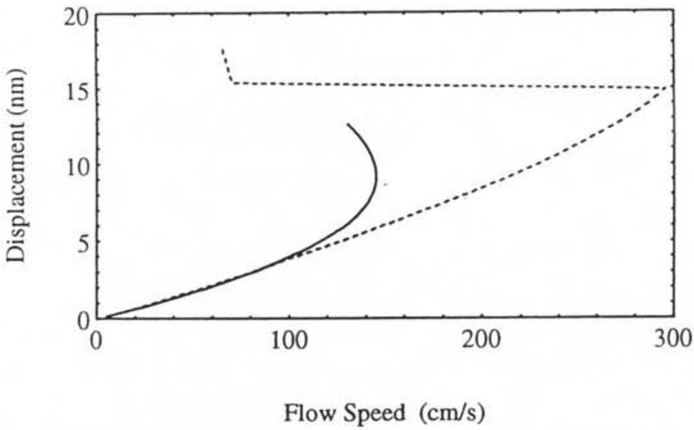

Fig. 2 Displacement vs. flow speed for a vortex bound to a Gaussian (solid) and circular (dashed) bump profile. The critical velocity corresponds to the maximum speed with a solution. Regions of negative slope are unstable.

Note that there is a point, corresponding to a maximum flow speed, where further displacements away from the bump are in equilibrium with a decreasing flow. These equilibria are unstable and the preceding maximum speed can be identified as a critical velocity for sweeping the vortex off the bump.

For small displacements (and flow speeds), the displacement is linear in the flow speed. The slope is another measure of the vortex pinning character on the bump. The reciprocal of the slope has units of frequency, an insightful way to think about it is roughly as a precession frequency for small amplitude motion about an azimuthally symmetric bump with the same profile. In this case, the vortex would be moving through the fluid without a background flow.

3. RESULTS AND DISCUSSION

Results are presented for Gaussian bumps of the form

$$f(x) = a\exp\left(-\frac{x^2}{b^2}\right) \tag{5}$$

and for circular bumps, parameterized by the same a and b so that the peak of the bump has the same height a above the y=0 plane and the same radius of curvature, $R_c = b^2/2a$.

$$f(x) = a - R_c + \sqrt{R_c^{\,2} - x^2} \qquad f(x) > 0 \tag{6}$$

This shape was chosen to intersect the y=0 plane with a kink as a contrast to the smooth transition to the flat plane that the Gaussian presents. For these two bump profiles, Fig. 3a shows the critical velocity for the Gaussian and the circular cases. Data are shown for heights a=3nm and several widths b as a function of film thickness. In both cases, varying the height a leads to a roughly proportional change in the critical velocities, and is not shown.

The critical velocity increases rapidly as b decreases for the Gaussian, especially in thin films, and has a much weaker dependence in the circular case. Here the kink dominates the critical behavior and also contributes to overall higher values since the vortices must be forced through a region of rapidly changing boundary curvatures (see Fig. 1).

The precession frequencies, shown in Fig. 3b for the Gaussian and circular cases, show a dramatic reversal of the critical velocity behavior. Recall that this quantity is a measure of the rigidity of the vortex against flow induced displacements near the peak of the bumps. The Gaussian bump precession frequency fairly well

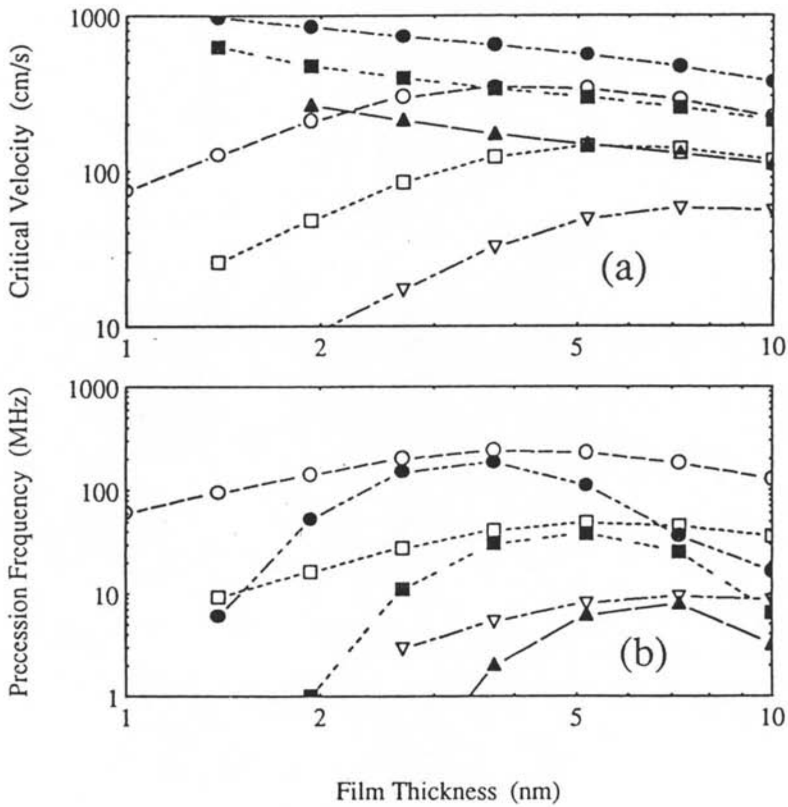

Fig. 3 Critical velocity (a) and precession frequency (b) for Gaussian (hollow) and circular (solid) bumps. The bump heights are 3nm and widths are 7.5nm (circle), 15nm (square) and 30nm (triangle). Both quantities roughly scale with the height.

reproduces the trends in the critical velocity, whereas the circular bump has changed to being much more sensitive to the width. As the film thins over the circle, the vortex looses its sense of where the peak is and responds to much smaller flow speeds. As before, the dependence on the height a is roughly proportional in both cases and not shown.

In considering the dynamical behavior of a vortex in response to AC flow, it is important to consider the equilibrium times over which one might expect the states discussed above to settle down. Presumably the precession frequencies of Fig. 4 represent the lowest frequency associated with motion of the whole vortex. Bends and undulations of more complex motion, necessarily involving sharper distortions of the vortex, would be higher in frequency and damp out faster, being closer in frequency to other excitations such as phonons in the film or ripplons in the free surface. Given the overall frequency scale of Fig 3b, it seems reasonable that the equilibrium is a fair description of the response of a vortex to the A.C. flow present in a third sound wave, for example, in the kHz range.

4. CONCLUSIONS

In all cases, the sharpness of the substrate features seem to dictate the pinning properties. Considering the trend seen in Fig. 3, it is probable that atomic sized features dominate the pinning. Decreasing the overall size leads to an increase in the critical velocity. It is thus expected that even in the presence of larger and broader substrate features the smaller ones, having the higher critical velocities, will dominate. Unfortunately, it is with the smaller, atomic sized features that the model discussed in this paper will break down due to the atomic vortex core size. The results, however, provide a basis for the discussion of vortex properties on real substrates which can have surface roughness on many different length scales.

It would be interesting to compare these results to a more hydrodynamically accurate solution to this problem. The effect of the core size, as well as a more realistic treatment of the substrate boundary and free surface should be investigated.

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REFERENCES

1. D. A. Browne, and S. Doniach, *Phys. Rev.*, **B25**, 136 (1982)
2. F. M. Ellis and L. Li, *Phys. Rev. Lett.*, **71**, 1577 (1993)
3. K. W. Schwarz, *Phys. Rev.*, **B31**, 5782 (1985)