Journal of Low Temperature Physics, Vol. 134, Nos. 1/2, January 2004 (© 2004)

Chaotic Third Sound Resonances

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There are three independent phenomena that compete to determine the line shapes of third sound resonances in a circular cavity. First, anharmonic terms in the hydrodynamic equations of motion lead to the usual hysteresis on a multi-valued response function. Second, wave coupling to vortices pinned in the film modify the resonant frequency as changes in the persistent current are induced. Finally, nonlinear dissipation can lead to saturation. The first two of these have been observed to result in continuous (not just transient) temporal behavior of the resonance amplitude with a fixed drive. Both cyclical and chaotic behaviors have been observed. The effects are dependent on the ability of the driven wave to either accelerate or decelerate the persistent current onto different amplitude branches of the multi-valued resonance.

PACS numbers: 67.40 Hf, 67.40 Vs.

1. INTRODUCTION

Third sound in superfluid ^{4}He films is remarkable for extremely large lateral film flow achievable as part of its oscillatory motion. The microscopic size of a typical film thickness (on the order of nanometers) combined with the macroscopic wavelength of the typical experimental geometry (on the order of centimeters) leads to lateral oscillations in the film that are larger than the thickness oscillations by the ratio of these two size scales: typically on the order of 10^7. That the film flows with little dissipation under such circumstances is a classic illustration of super-flow.

The extreme flow does not occur without consequence. Several nonlinear phenomena are associated with third sound[1–3] and this paper reports on observations of third sound resonances which, although driven with a steady oscillatory force, respond in an unstable, possibly chaotic manner.

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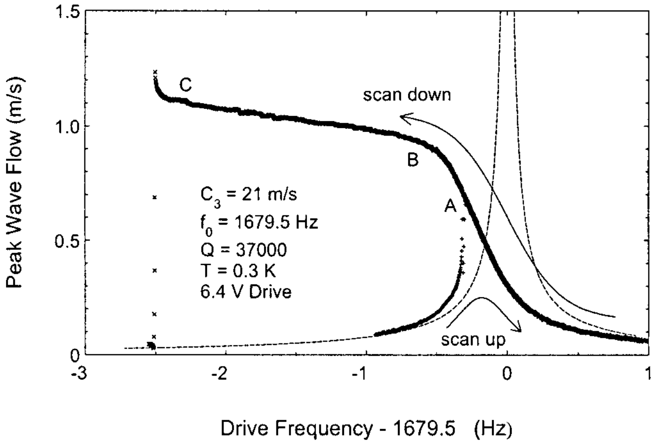

Fig. 1. Third sound resonance scanned first up then down. The mode coupling shift dominates at low amplitude (below point B) and the circulation shift takes over at higher amplitudes (above point B). The dashed line shows what the linear response would be in the absence of any shifts. The down scan, taken after the up scan, resulted in a permanent shift of the resonance location down by 0.72 Hz.

The nonlinear effects that come into play are best illustrated by the features of the scanned third sound resonance shown in Fig. 1. The resonator used is circular, 12.3 mm in diameter, with the modes driven and detected electrostatically. This allows for an unambiguous calibration of amplitude, which henceforth will be referred to as "peak wave flow". This is the magnitude of largest oscillatory flow occurring within the resonator. For the $m = 2$ mode used, $v_p = 0.360 c_3 \frac{\eta}{h}$, where c_3 is the speed of third sound (21 m/s throughout this paper), η is the height displacement of the $m = 2$ Bessel function describing the mode, and h is the film thickness. The numerical factor of 0.360 comes from the point of maximum flow, occurring at approximately 3/4 of the resonator's radius. Note that the peak flow of around 1 m/s is quite extraordinary given the 2.6 nm film thickness.

The normally degenerate circular modes are split by any D.C. circulation in the resonator.[2] The resonance shown in Fig. 1 was begun in the presence of a significant circulation of about 17 cm/s, and is Doppler shifted up from the degenerate position by about 16 Hz. The rotating wave of this up-shifted mode travels in the direction of the circulating flow. The frequency axis Fig. 1 is set to zero at this point.

The dashed line represents the expected line-shape in the absence of nonlinear effects. As the resonance is approached from either direction, resulting in a significant amplitude, the first noticeable deviation from the linear response is a reversible shift to lower frequencies. This shift is initially proportional to the square of the amplitude and is primarily due to inter-mode coupling mediated by the Bernoulli pressure.[3] This effect skews the linear response curve dramatically to the left, forming two possible steady state conditions for frequencies to the left of point A. In the vicinity of this point, the up-scan jumps from the below-resonance branch up to the above-resonance branch of the skewed response. High amplitudes of the on-resonant condition are bypassed by this jump.

As the drive frequency is scanned down, the response again reaches the vicinity of point A on the above-resonance branch, and continues past. At approximately $v_p = 0.9$ m/s, the oscillatory wave flow is enough to de-pin the vortices responsible for the circulation.[4] A decay in the circulation is induced. The line-shape is drawn out even further, this time by an irreversible shift down in the frequency, following the direction of the scan, and reflecting the induced decay. When point C is reached, the wave flow amplitude has grown large enough to induce a forward circulation, or "swirl" the film up, through vortex creation and redistribution.[4] The subsequent Doppler induced mode up-shift rapidly moves the wave conditions through resonance in a runaway fashion, catastrophically jumping back to the tail of the below-resonance branch. Although the Doppler shift decayed approximately 2 Hz during the down scan, the net effect of the violent self-scan through resonance was to send the circulation (and resonance) back up. After the scan sequence shown in Fig. 1, the low amplitude resonance point ended up at 0.72 Hz below where it began.

If the circulation were not changed, the up and down scans would follow a repeatable path along he above- and below- resonance branches of the response distorted by the coupling shift. These down-shifted branches allow a hysteresis in the up and down scans, that are similar, but opposite in sign to those on a driven whirling string.[5] The possibility of circulation changes complicates the behavior, as discussed. Depending on the exact drive conditions and the temperature, the amplitude along either of the branches could swirl up the circulation or induce it to decay. The up scan of Fig. 1 did neither: conditions were specifically chosen so that it could be scanned first, leaving the resonance alone. Both the up and down scans shared part of the same above-resonant response.

2. OBSERVATIONS AND DISCUSSION

The above effects allow for an interesting phenomenon: with a fixed drive, it is possible to begin initially below resonance, but at an amplitude

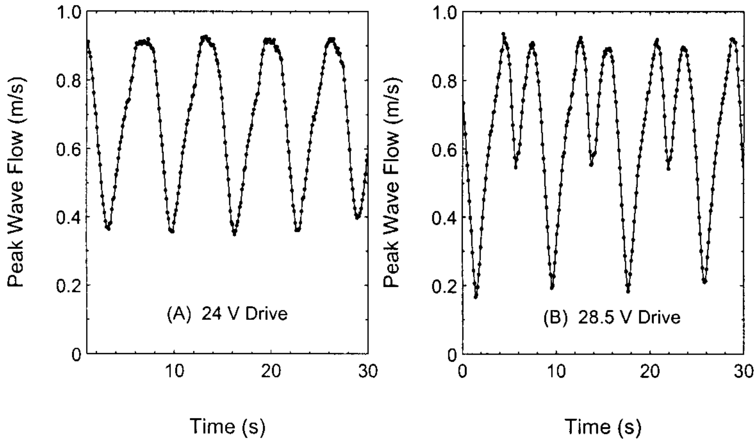

Fig. 2. Cyclically astable third sound resonance at 0.6 K with a 24 V drive (A) and 28.5 V drive (B). With a fixed drive frequency, the third sound amplitude cycles between the below-, and above-resonant branches of the hysteresis in Fig. 1.

large enough to induce decay through vortex de-pinning. The diminishing circulation will shift the mode *into* resonance, jumping to the above-resonant branch. On this branch, the amplitude is larger, and capable of swirling the circulation, sending the mode back up. The mode drops back down to the below resonant branch, and the cycle repeats. This behavior is demonstrated by the data in Fig. 2. Many thousands of oscillatory cycles occur over the course of the resulting cyclical amplitude modulations, shown at two different drive levels.

These cyclical modulations are somewhat delicate and require conditions with a scope that is not yet surveyed. The behavior reported here was observed in the $m = 2$, $n = 1$ mode at 0.6 K where the right combination mode coupling shift, circulation shift, and hysteresis coincide with a favorable thermally enhanced induced decay rate. The range of stable cyclical behavior with drive is typically small, stable only for drive frequencies within a 0.1% range. Near the boundary of these conditions the cyclical behavior can degenerate into a chaotic-like modulation. An example is shown in Fig. 3, taken at 0.5 K with a 30 V drive.

It is difficult to prove that an observed phenomenon is chaotic, particularly when dynamical noise is present and the data set is temporally limited. Further investigation of the qualitative aspects of the resonances show that chaos is plausible. Fig. 4 shows the cyclical magnitude behavior previously shown in Fig. 2 as complex amplitude plots. In (A), the complex amplitude,

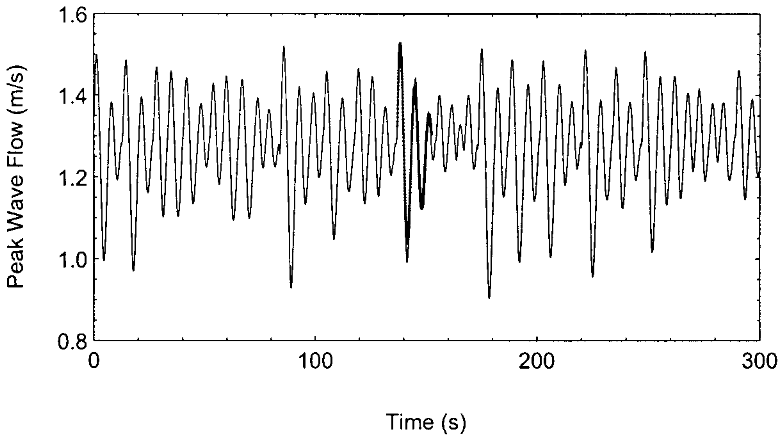

Fig. 3. Chaotically astable resonance at 0.5 K and 30 V fixed drive. Cycling around the resonance hysteresis results in sections of transient decay beats, one of which is highlighted.

referenced to the fixed drive, cycles counter-clockwise in time around the graphs indicating the wave oscillations are slightly higher than the drive. This corresponds to a drive on the the below-resonant branch of the scanned case. The wave is inducing the circulation to decay into resonance. The amplitude builds up when the phase (relative to the drive force) gets close to -90^o. The "retro" motion is the jump to the above-resonant branch (wave oscillations *below* the drive) where the circulation, and resonant frequency increase, returning to the below-resonant branch. It should be mentioned that this cyclical behavior is far from the steady state condition assumed in a scan such as in Fig. 1. The phase advance of one cycle demonstrates that these trajectories are dominated by transient oscillations. References to the steady state response can only be qualitative.

In Fig. 4 (B), the wave advances two cycles relative to the drive before hopping to the above-resonant branch. Although the trajectory seems to retrace itself after one cycle, the circulation (which is obtained from the trajectory rate) is different on alternate passages.

These graphs indicate the possible basis for a chaotic modulation of the response. There are two dynamical variables in effect: the (complex) amplitude, and the circulation. The jump point to the above-resonant branch appears to be associated with hyperbolic point[5] in the phase space of these two variables. The amplitude of the chaotic example (Fig. 3) shows occurrences of one, two, and more transient decay "beats" terminating with close encounters to the hyperbolic point. One such section of about two beats is

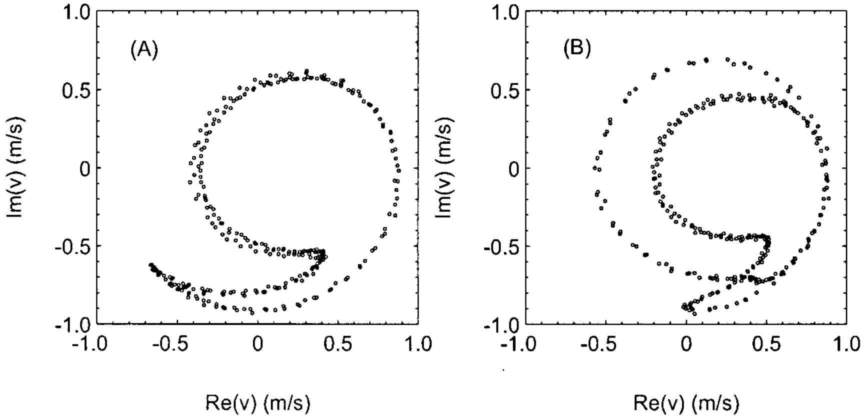

Fig. 4. Real and imaginary parts of the cyclical behavior of Fig. 2. The wave cycles CCW around the plots, gaining one cycle of oscillation relative to the drive for each round trip.

highlighted near the middle of Fig. 3.

3. CONCLUSIONS

Third sound resonances have been observed to display highly nonlinear behavior involving line-shape distortions due to mode coupling and swirling. The mode coupling shifts provide a hysteresis to the line-shape, and the vortex de-pinning and nucleation induce circulation changes that can shift the resonance either way. The precise conditions under which these effects allow for cyclical, quasi-periodic, or possibly chaotic amplitude modulation are yet to be determined, but the mechanism responsible for the instability has been identified. Numerical simulations of the mechanism are presently being investigated.

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