Third sound dissipation at a point contact

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Abstract. Third sound on a planar geometry at low temperatures is characterized by a rapidly diminishing thermal dissipation. Direct mechanical dissipation is limited to that associated with defects in the system. This includes interaction with pinned vortices, critical flow at surface defect sites, and unintentional acoustic coupling. Dissipation of this latter type is possible in the parallel plate geometry of capacitively detected third sound. We calculate the coupling of a third sound wave across a contacting bridge to a parallel plane, and investigate the energy transfer out of the wave and flow properties of the film in the vicinity of the contact. The presence of various mirror waves on the contacting plane is also considered. Experimental dissipation is observed in both geometries and it is shown that a single contact is capable of accounting for the dissipation as well as an unusually low observed critical velocity.

1. Introduction

A common geometry for the study of third sound involves a parallel plate in close proximity to the superfluid film within which the third sound propagates. Resonators involving a closed cavity [1], capacitive thickness detection [2], or gas confinement for the study of fifth sound [3] are examples. It is often the case that both plates participate in the wave motion, as with a closed resonator. Indeed, it has been established that wave propagation in the gas is capable of coupling the third sound energy from one plate to the other [2], and the detailed equations of motion for such a coupled wave have been presented. [4]. We have been developing a capacitively detected resonator where the detection electrode is not an inherent part of the resonator surface. One of our goals is the prevention of coupling between the plates of the third sound. This requires a low temperature to eliminate the vapor coupling, but it also requires the absence of any physical contact between the plates. The presence of small physical contacts has not been an issue with previous resonators where the intended wave is identical on opposing plates. The quality factors in such resonators can be quite high.

With this paper, we present our analysis of the consequences of a small contact between two plates with different wave states on each plate. We first examine the communication of a plane wave across an idealized bridge, and then apply the results to various resonator geometries. Two effects can then be quantified: (1) the transfer of energy between the plates, and (2) the concentration of flow at the contact point. The first has obvious consequences for the quality factor of a resonator with such a contact, and the second will be capable of producing critical flow well before it occurs in the intended wave field.

2. Contact transmission derivation

We wish to obtain a result of use that is independent of any specific resonator, so we start by considering the considering an idealized bridge between two infinite, parallel planes, one of which has a plane wave propagating imposed at infinity. Figure (1) shows the idealized geometry where the point contact is taken to be a cylindrical bridge between the two planes. This is a scattering problem which

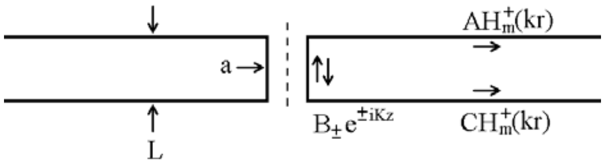

Figure 1. Idealized contact point. The ambient wave is on the top plane. Only the scattered components are shown. The contact radius is *a* and the length is *L*.

we solve exactly for simplified equations of motion of third sound. The problem is dominated by mechanical fluid coupling, so we take T=0, ρ_n=0, and $\rho_s=\rho$. The linearized equations of motion for the height displacement η and superfluid velocity $\vec{v}$ are

$$\frac{\partial \eta}{\partial t} = -h\vec{\nabla}\cdot\vec{v} \quad \text{and} \quad \frac{\partial \vec{v}}{\partial t} = -g\vec{\nabla}\eta \tag{1}$$

where the vectors are restricted to the surfaces and g is the van der Waals force per mass at the film's surface. The solutions can be expressed in terms of scalar amplitudes η decomposed into azimuthal components associated with angular functions $\exp(im\phi)$. The scattered cylindrical Hankle function components of figure (1) are included along with the Hankle components of the imposed plane wave, taken to be traveling in the ϕ=0 direction on the top surface. The bridging cylinder has solutions $\exp\left(\pm i\sqrt{k^2 - \left(m/a\right)^2}\, z\right)$ for wavenumber k. The wave components are matched in height and normal slope at the boundaries, and an exact result for the scattered wave components is straight-forward, but uninspiring. Our interest is in the limit of small contact size compared to the wavelength. The solutions can be expanded to lowest order in ka and kL. For a unit amplitude plane wave in this limit, the scattered wave components are:

$$A = 0 \quad \text{and} \quad C = \frac{2\pi i^{m+1} e^{-mkL}}{m!(m-1)!}\left(\frac{ka}{2}\right)^{2m} \quad \text{for} \quad m \neq 0 \tag{2}$$

$$A = -C \quad \text{and} \quad C = \frac{\pi i}{2\left(\pi i + kL - 2\left(\ln(\frac{ka}{2}) + \gamma\right)\right)} \quad \text{for} \quad m = 0 \tag{3}$$

where γ is Euler's constant. The incident plane wave induces equal $m = 0$ scattered components on both surfaces, and only significantly excites $m \neq 0$ components on the opposite surface. Applying these results to the standing waves $\eta\cos(kx)$ and $\eta\sin(kx)$ yields radiated powers

$$W_{\cos} = \frac{W_0}{\pi^2 + \left(kL - 2\left(\ln(\frac{ka}{2}) + \gamma\right)\right)^2} \quad \text{and} \quad W_{\sin} = \frac{W_0}{2}(ka)^4 e^{-2kL} \quad \text{with} \quad W_0 = \frac{\pi^2 \rho h c^3}{2k}\left(\frac{\eta}{h}\right)^2 \tag{4}$$

respectively, and $c = \sqrt{gh}$. The $m = 0$ adoption to the standing wave form requires careful consideration of the small ka and kL expansions. The cosine wave, with an antinode at the position of the contact, radiates $m = 0$ waves on both plates and the sine wave, with a line node traversing the contact, radiates an $m = 1$ wave on the opposite plate. These results constitute the basis of the scattering results that can now be used to analyze the properties of a small contact to the surface of a third sound resonator.

3. Application to a resonator

We shall now consider the situation where a third sound resonator has an active surface (part of the intended resonator) that is bridged to an inactive surface (not part of the intended resonator) by the previously discussed contact. There are two features of the contact that are relevant. First, the energy losses from equations (4) can be used to estimate inherent loss factor Q^{-1} of the contact. Second, the induced wave amplitudes from equations (2) and (3) can be used to obtain the local superfluid flow at the contact.

The position of the contact relative to the nodal structure of the resonator will determine the appropriate local amplitude to be used. The local height oscillation at the contact is what drives the $m = 0$ radiated wave and the local flow oscillation past the contact is what drives the $m = 1$ radiated wave. These quantities are dependent on the particular geometry of the resonator, but to keep the discussion as general as possible, we will take a square wavelength as the resonator area and a phase angle ϕ to characterize the position of the contact, with $\phi = 0$ on an antinode and $\phi = \frac{\pi}{2}$ on a node. The loss factor is then found to be

$$Q^{-1} = \frac{\frac{1}{2} W_{\cos} \cos^2 \phi + W_{\sin} \sin^2 \phi}{W_0} \tag{5}$$

which, with equations (4), gives a simple expression for the loss associated with the contact in terms of the contact geometry and position. The factor of $\frac{1}{2}$ on the cosine term results from an assumption that the $m = 0$ wave on the resonator surface does not contribute to any loss. Figures (2) and (3) show some values of this loss factor as a function of contact size *ka*.

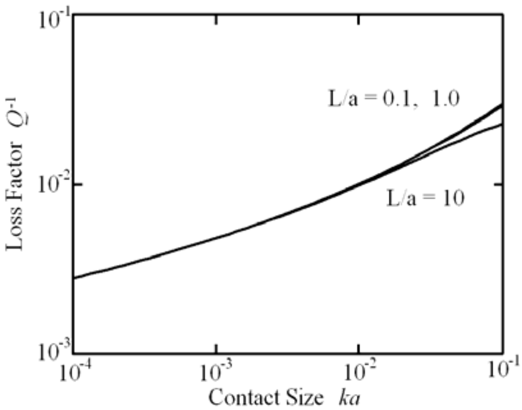

Figure 2. Loss factor for the *m*=0 case.

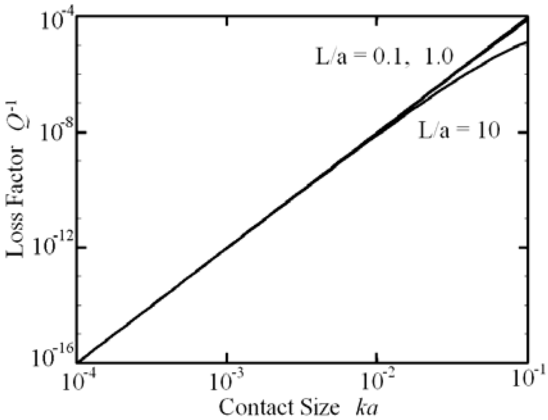

Figure 3. Loss factor for the *m*=1 case.

The $m = 0$ losses are clearly the most offensive for all but the degenerate situation where the contact happens to lie on the node. This situation cannot be dismissed: in a resonator of high symmetry such as a circular resonator, it is possible that a degenerate mode would be split by the presence of the contact into modes with node and antinode orientations at the contact.

Turning to the flow velocity at the contact point, the $m = 0$ case is the only one of interest. The $m = 1$ coupling assures the continuity of ambient film flow through the contact, and is therefore matching the local flow. The $m = 0$ induced flow is a new feature arising from the continuity of the surface profile at the contact. This induced flow can be quite large since the small contact is required to supply fluid necessary to distort the film profile with an extent much larger than the contact. This requirement, consistent with the equations of motion, is the origin of the logarithmic terms associated with the $m = 0$ Bessel functions of the second kind. The flow through the contact is found from equation (3) along with the asymptotic form of the corresponding Hankle function:

$$v_{contact} = c \frac{\eta}{h} \left[ka \left(\pi i + kL - 2 \left(\ln(\tfrac{ka}{2}) + \gamma \right) \right) \right]^{-1} \tag{6}$$

The last factor on the right is an enhancement ratio for comparison to the ambient wave film flow. Figure (4) shows this factor for the contact sizes of figures (2) and (3). It can be quite large, illustrating how small contacts are capable of generating large flow speeds. Note that such contacts are where the flow will first reach critical conditions.

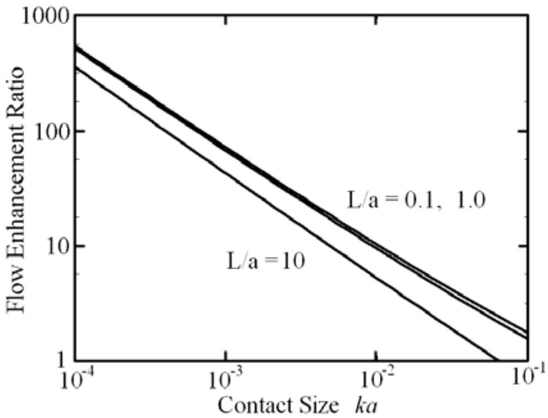

Figure 4. Magnitude of the film flow enhancement ratio term of equation (6). This is the ratio of the film flow velocity through the contact to that in the surrounding third sound wave.

These results were for the case of a contact between an active surface and an inactive surface. If the contact is between two active surfaces of a resonator with different amplitudes or phases, there will be no radiative loss within the context of this analysis. The mode functions will be modified by terms similar to the radiative functions in the vicinity of the contact, but the energy is redirected within the mode. The flow amplitude at the contact, however, will include the same enhancement factor, but the amplitude term will be modified by the replacement $\eta \rightarrow \eta_1 - \eta_2$, the complex amplitude difference. It is important to note that since the enhancement factor can be very large, small asymmetries in a face-to-face closed resonator can be the source of unexpected super-critical flow if there is a contact.

Although we have not performed a systematic experimental study, we have observed many cases of anomalously large damping, with Q in the range of 300 - 3000 in resonators shown by other means to have contacts. Such damping is consistent with even a single point contact of a size that might have been overlooked, such as a 10 μm particle touching between the resonator surface and the nearby capacitive electrode plate. These damping cases have always been accompanied by amplitude dependence at correspondingly small flow speeds confirming the presence of flow conc3entration.

4. Conclusions

We have presented results of a calculation describing the behavior of an idealized point contact bridging the gap between two surfaces supporting a third sound wave. The resultant damping is most egregious in the case of one active surface and is dominated by a driven flow across the bridge. Even very small contacts are capable of completely overwhelming other forms of dissipation in a typical resonator. We have also quantified the extent of the flow concentration at the contact, demonstrating how such contacts are capable of exceeding a critical flow condition well before that achieved by the ambient wave flow. Both effects depend primarily on the perimeter of the contact with only a weak dependence on the length.

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