

Surface excitations and partial entropy of third sound in ^4He

Y Pramudya and F M Ellis

Department of Physics, Wesleyan University, Middletown, CT 06459 USA

E-mail: ypramudya@wesleyan.edu

Abstract. The hydrodynamics of third sound in superfluid ^4He films passes through a regime dominated by surface excitations as the temperature is lowered. With a limited spatial extent, such excitations decouple from the thermomechanical forces operating within the context of the two-fluid model. This phenomenon is the basis of the “partial entropy” of Bergman’s classic development of third sound hydrodynamics [1], but its distinction from entropy has been lost by subsequent authors. We present a computational example of its hydrodynamic significance and discuss the consequences for the thermomechanical aspects of third sound.

1. Introduction

Third Sound is an acoustic mode within two fluid hydrodynamics where the motion is dominated by the flow of the superfluid component along the substrate of an adsorbed film. The intimate contact with the substrate locks the normal component to $v_n = 0$, and the free surface relieves the pressure of compression. Like second sound, third sound has a mechanical component (kinetic energy and van der Waals potential energy) as well as a thermal component associated with the concentration and dilution of thermal excitations. Both sounds exhibit propagating thermal fluctuations, but the nature of the thermal excitations [2][3] and constraint on their mechanical coupling give the thermomechanical effects within the film some unexpected behavior.

We begin by describing the hydrodynamic equations of motion for third sound and illuminating the subtlety of Bergman’s “partial entropy” [1]. We then present results of a numerical computation of the partial entropy for a simplified set of excitations [2] for the purposes of illustrating its distinction from a generic “film entropy” which pervaded the subsequent third sound literature [4]. Variational approaches to third sound [5][6] correctly include the physics, but as the understanding of film excitations improves [3], we re-emphasize the hydrodynamic consequence of partial entropy in the context of the two fluid model.

2. Third sound equations of motion

The thorough development by Bergman remains the basis used by those with experimental access to the hydrodynamic variables of the two-fluid model. Our emphasis here is on the concept of partial entropy, and as a consequence, we express his equations in a form that more intuitively separates the various terms according to their function. We also work in one dimension with all variables appropriately film-averaged. We take $v_n = 0$, with η , τ , and v the oscillating thickness,

temperature, and superflow, respectively :

$$\rho \frac{\partial \eta}{\partial t} = -\rho_s h \frac{\partial v}{\partial x} + J_m \quad (1)$$

$$\rho h C \frac{\partial \tau}{\partial t} = \rho_s h T \bar{S} \frac{\partial v}{\partial x} - L J_m - J_Q \quad (2)$$

$$\frac{\partial v}{\partial t} = -g \frac{\partial h}{\partial x} + \bar{S} \nabla \tau \quad (3)$$

The first expresses conservation of mass in the incompressible of film thickness h including superflow, $\rho_s v$, and a mass flux from the film into the vapor, J_m . Equation (2) expresses conservation of thermal energy derived from the consideration of entropy. Without any normal component motion, entropy can be diluted by superflow, or otherwise changed by heat flux through evaporative latent heat, $L J_m$, and conduction, J_Q . The last equation expresses conservation of mechanical energy in the form of Newton's law. The van der Waals acceleration at the film surface is g , and the thermomechanical force involves the thermal gradient.

Bergman's inclusion of L distributed among several terms is not very transparent. It results from a careful derivation from the two fluid model including motion of the normal component. Once this connection is lost with the $v_n = 0$ approximation, it is more transparent to put the L into the single evaporative term in equation (2). The partial entropy per mass denoted by \bar{S} in both equations, comes about through consideration of a fixed areal patch looking down perpendicularly upon the film. In equation (2), for example, the total entropy per area, assumed to be a state variable dependent only on thickness and temperature, $\Sigma(h, T)$, has two corresponding partial derivatives responsible for the film thermodynamics functions arising from its derivative:

$$\left(\frac{\partial \Sigma}{\partial T} \right)_h = \frac{\rho h C}{T}, \quad \left(\frac{\partial \Sigma}{\partial h} \right)_T = \rho \bar{S} \quad (4)$$

Here, C is the film heat capacity per mass, but \bar{S} , rather than "film averaged entropy", is a new quantity combining all the effects of a changing film thickness into the dilution of entropy. A similar consideration produces the partial entropy in equation (3). In terms of the more intuitive film entropy per mass, S , the partial entropy is

$$\bar{S} = \frac{\partial(hS)}{\partial h}. \quad (5)$$

This form for the entropy is explicitly pointed out as separate from the other film averaged quantities in Bergman's derivation, but seems not to have been properly passed on by subsequent authors. At low temperatures, the difference between the film entropy and the partial entropy can be significant. The differences result from the nature of the thermal excitations that dominate the normal fluid component, and depend strongly on both thickness and temperature. We compute these differences for our model system in the next section.

The solutions to the equations of motion are rather involved with the detailed substrate and vapor interactions included for the geometry of interest. For the purposes of illustrating how the partial entropy comes in, it suffices to say that thermomechanical terms appear in two primary forms associated with the adiabatic limit. First, the temperature oscillations in the adiabatic film, given by $\frac{\tau}{T} = -\frac{\bar{S}}{C} \frac{\eta}{h}$, and second, the magnitude of the thermomechanical restoring force expressed as $c_h^2 = \frac{\rho_s}{\rho} \frac{T \bar{S}^2}{C}$. The latter is the fifth sound speed [4] [7] with the partial entropy substituted. The influence of each of these terms is reduced (and phase shifted) as a result of vapor and substrate coupling as the temperature rises, but the presence of the partial entropy remains linked to these terms.

3. Film thermodynamics

To illustrate the distinction of partial entropy from film entropy, we use a simplified excitation spectrum $\omega(q)$ valid for films not too thin ($h > 1$ nm) and not too hot ($T < 0.75$ K) that includes : (1)The surface wave mode (third sound/ripplon spectrum) with $\omega_{surf}^2 = \tanh(qh) \left(gq + \frac{\gamma q^3}{\rho} \right)$; (2)bulk-like film phonon branches with $\omega_n^2 = c_1^2 \left(\left(\frac{(n-1/2)\pi}{h} \right)^2 + q^2 \right)$; and (3)bulk rotons for the purposes of connecting to the high temperature, thick film limit. Layer modes [3] have not been included for simplicity, but would be important in thinner films. The three spectra were integrated numerically with the Planck distribution to obtain the thermodynamic functions for the film specific entropy $S(T)$ and specific heat $C(T)$. The partial entropy was evaluated using equation (5). A sufficient number of phonon modes were included to assure quantitative accuracy for $T < 0.75$ K. Figure (1) and figure (2) show the effect of the partial entropy for a range of film thickness expressed in terms of quantities relevant to the third sound equations of motion.

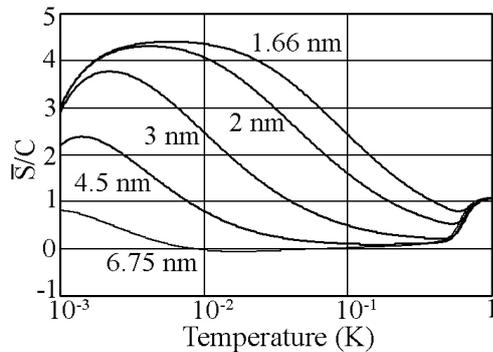


Figure 1. Ratio of the partial entropy to the specific heat giving the adiabatic temperature oscillations for third sound.

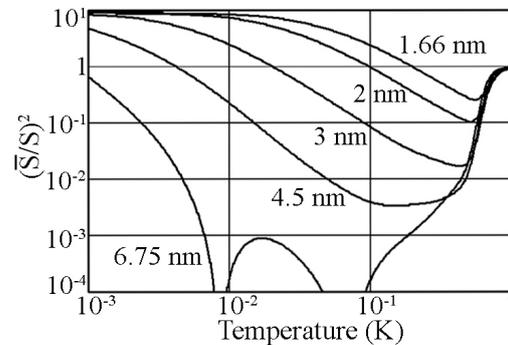


Figure 2. Ratio squared of the partial entropy to the film entropy representing the effect on the thermomechanical force.

4. Discussion

The partial entropy emerges from the two fluid model applied to films due to a unique combination of the mechanical constraint imposed by the free surface and the thickness dependence of the excitations themselves. As the volume available to excitations expands or contracts with the movement of the free surface, the nature of the excitations involved dictates the character of thermal response. Three idealized cases can help clarify this concept, again taken in the adiabatic sound limit and with the simplified excitation spectrum to emphasize the role of the partial entropy. Consider first the case where the film is taken as a slab of bulk as would be expected for thick films at high temperatures. Since the entropy per area grows proportional to thickness, the partial entropy per mass from equation (5) is just the bulk entropy per mass, $\bar{S} = S_{bulk}$. As mass and heat move around in the film, the intensive quality of the thermal excitations as the normal component dominates. This is perhaps the historic reference frame that allowed film entropy to displace partial entropy in the literature.

Next, consider low temperatures where the very long-wave free-surface excitations are populated. This is the third sound limit where capillarity is negligible compared to the van der Waals force. With $\omega = \sqrt{g(h)h}q$, the thermodynamics can be analytically determined. The partial entropy with $g(h) \propto h^{-4}$ is $\bar{S} = 3S$. This limit has a partial entropy significantly greater than the film entropy. Converging superfluid at a wave crest not only dilutes the

normal component, but softens the excitation spectrum as the film thickens. The partial entropy expresses this constraint analogous to adiabatic demagnetization.

Finally, consider the case where *all* of the thermal energy is associated with surface excitations having no connection to the thickness. This would be the case of moderate temperature and thickness dominated by high- q ripples. Here, the entropy per area is independent of h and the partial entropy, from equation (5), is $\bar{S} = 0$. This would also apply to the layer modes.

All three cases are visible in the numeric data of figures (1) and (2). At low temperature, all thicknesses approach the $\bar{S} = 3S$, third sound limit. At high temperatures, the dominance of the phonon branches and rotons give the entropy a bulk-like thickness dependence, approaching $\bar{S} = S$. Most interesting is the intermediate range where the surface ripples dominate. This begins around 0.5 K and extends down to where the ripples begin to sense the substrate – the third sound modes – with thicker films approaching $\bar{S} = 0$ over a widening temperature range.

Surprisingly, the partial entropy for films with $h > 6.1$ nm has a negative partial entropy. Figure (3) maps this range of thickness and temperature for our simplified excitation spectrum. Ripples with $qh < \sim 1$ and $\frac{\gamma q^3}{\rho} > \sim gq$ have thickness dependence favorable for the mode softening. These conditions, with $T = \frac{\hbar\omega(q)}{k}$ are shown in figure (3) as the shaded region. Note that here, both the thermomechanical force and the temperature oscillations in third sound would be reversed.

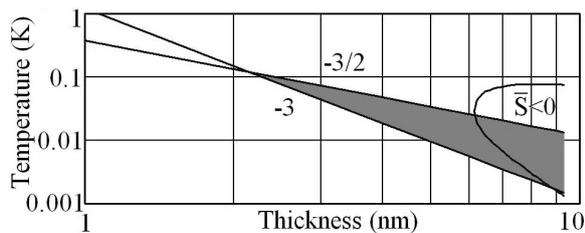


Figure 3. The region within the solid arc shows the range of negative partial entropy predicted by the simplified excitation spectrum. The shaded region is described in the text.

5. Conclusion

A proper treatment of partial entropy within two-fluid third sound hydrodynamics significantly alters the magnitude of thermal effects where surface excitations dominate the population of thermal excitations. The distinction between film entropy and the original partial entropy of Bergman must be recognized. The partial entropy is smaller than the film entropy for intermediate thicknesses and temperatures, and larger than the film entropy for all thicknesses at very low temperatures. These conclusions are based on the qualitative character of the excitations and are not limited to our simplified spectrum.

The region of negative partial entropy at intermediate thickness and temperature is, however, susceptible to the details of the excitation spectrum. Its occurrence in the the simple film-ripple spectrum illustrates how the reversal of thermomechanical attributes of the two-fluid model could occur in films. In spite of the reversal, the sign of the modified fifth sound speed remains positive under all conditions.

References

- [1] Bergman D 1969 *Phys. Rev.* **188** 370
- [2] Saam W F and Cole M W 1975 *Phys. Rev. B* **11** 1086
- [3] C E Campbell B E Clements E K and Saarela M *Phys. Rev. B* **55** 3769
- [4] Putterman S 1974 *Superfluid Hydrodynamics* (North-Holland)
- [5] J E Rutledge W L M and Mochel J M *Phys. Rev. B* **18** 2155
- [6] Krotscheck E and Miller M D *Phys. Rev. B* **73** 134514
- [7] I Rudnick J Maynard G W and Putterman S 1979 *Phys. Rev. B* **20** 1934