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Observation of the persistent-current splitting of a third-sound resonator

F. M. Ellis and Hai Luo
Department of Physics, Wesleyan University, Middletown, Connecticut 06457
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The splitting of third-sound modes in a circular resonator due to the presence of a moderate-velocity persistent current has been observed. The absence of splittings in previous resonators is explained in terms of the dynamics of a two-level system. Single quanta of circulation should be observable in a smaller resonator.

Third sound[1] is a mode of wave propagation in thin superfluid ^{4}He films analogous to long-wavelength gravitation waves in water where the van der Waals attraction to the substrate on which the film is adsorbed takes the place of gravity. Unlike gravitational waves, the third-sound height oscillations are accompanied by significant temperature oscillations. Doppler-shifted third-sound pulses have served for years as a probe of the flow states in these films. The study of the decay of persistent currents[2] and critical velocities[3] are classic examples. Here, a thermal drive transducer is flanked by pickup bolometers upstream and downstream from a drive heater and the difference in arrival times of a third-sound pulse determines the film flow speed. The development of third-sound resonators[4,5] greatly improved the potential precision of these measurements because of the intrinsically superior accuracy of resonance techniques over time-of-flight measurements. In other applications, such as probing the thermodynamic or structural properties of films,[6,7] this has indeed been the case. Unfortunately, the closed and restricted geometry of resonators does not lend itself easily to persistent current applications. Earlier experiments incorporating third-sound resonators of various geometries on rotating cryostats have failed to show the expected splitting of resonant modes even when the presence of a film flow was evident by other means.[8,9] Persistent current splittings of other superfluid sound modes have been observed[10,11] but not in two-dimensional films.

In a flat circular resonator, the film is adsorbed onto the inner surface formed by two circular disks joined at their perimeter. The normal modes of the thickness oscillations of the third-sound wave are given by

$$\delta h(r,\phi)=h_0 J_m(x_{nm}r/a)e^{im\phi}\,, \quad (1)$$

where r and ϕ are polar coordinates, a the radius of the disk, and x_{nm} and m determine the mode configuration. For our resonator x_{nm} is a zero of the derivative of $J_m(x)$. The frequency is $\omega_0=C_3x_{nm}/a$ with C_3 the third-sound speed. The modes with m having opposite sign are degenerate if there is no steady-state film flow. These modes are analogous to traveling waves in the sense that the height profile rotates at a frequency ω_0 about the axis of symmetry. An equally acceptable basis can be formed from the sum and difference of modes (1) with opposite m. These are standing waves with stationary node lines. If there is a background film flow, the degeneracy may be lifted because of the Doppler shift. The resulting resonant frequency shifts for arbitrary resonator geometries and flow fields would be difficult to calculate but for small flow speeds one can use perturbation techniques. For the circular resonator with the curl free, symmetric flow field $v(r)=V_0(a/r)$, the shift for small V_0 can be written as

$$\delta\omega/\omega_0=\gamma V_0/C_3\,, \quad (2)$$

where γ accounts for the weighting of the Doppler shift within each mode and includes a factor of ρ_s/ρ, the superfluid fraction. The number γ depends on the mode approaching 1 from above as m increases. Modes rotating with the flow are shifted up in frequency and modes rotating against the flow (m of opposite sign) are shifted down. The quantum nature of superfluid films restricts V_0 to integral multiples of $\hbar/m_4a$ (Ref. 12) (m_4 the mass of a helium atom) reflecting the single valued nature of the superfluid wave function. Ideally, one would hope that the sensitivity of the resonator modes to V_0 would be enough to see changes in the circulation state of a single quantum.

The resonator used is similar to that described in Ref. 4 with several perhaps crucial differences. It consists of two silvered glass plates separated by a 9 μm gap glued together with stycast 1266 epoxy. The resonator is defined by a bubble in the epoxy whose outer edge was fixed to be a 1.17-cm diam circle by a small circular etch pit approximately 10-μm deep and 30-μm wide in one of the plates. Small holes (0.3-mm diam) through the center of the circle in each plate allow a slight pressure to be applied during the gluing process and sample access later. Both the driver and receiver transducers are electrostatic; the film will tend to be pulled into regions of electric field inside the drive region of the gap when a voltage is applied and changes in the film thickness in the pickup region will change its capacitance. Thus the drive and the pickup are directly coupled to the mechanical degrees of freedom in the third-sound modes. Both of the electrodes were isolated from the surrounding silver by the shadows cast by 50-μm wire masks during the silver evaporation. The electrodes were configured to couple efficiently with the $m=1$, $n=1$ mode. For this mode, $x_{nm}=1.841\ldots$ and $\gamma=1.277\ldots$. Copper foil glued to the outer surfaces of the glass plates provide both thermal and mechanical mounting inside of a small copper cell containing approximately 25 m^2 of alumina powder for film stability. A 0.13-mm inner-diameter capillary provided sample access to the cell. All data presented in this work were taken at 60 mK, well below the superfluid onset temperature.

An oscillating voltage up to 10 V_{pp} applied to the drive electrodes produces a well-defined force on the film at twice the drive frequency. The response of the film can be monitored as a modulation of the capacitance of the pickup electrodes. This capacitance is converted into an electrical signal by using it in an L-C tunnel diode oscillator circuit phase locked to a reference synthesizer. The phase error is the demodulated signal and is detected with a two-phase lock-in amplifier referenced to twice the drive frequency. As the drive frequency is varied, the film responds as a series of resonances, each one corresponding to modes given by (2).

Figure 1 shows the $n=1$, $m=\pm 1$ mode observed with a third-sound velocity of about 24 m/s. The film thickness is 7.1 atomic layers based on the third-sound velocity. This agrees with the thickness estimated via a Brunauer-Emmett-Teller isotherm[13] and shifts in the capacitance observed during sample addition. Shown are two different shifts of 0.14% and 0.37% corresponding from (2), with $\gamma=1.277\ldots$ for these modes, to a velocity at the perimeter V_0 of 2.7 cm/s in (a) and 6.8 cm/s in (b). In this case, between the two scans shown, the experiment was warmed to 4 K, part of the sample was removed, then cooled back down and the sample rapidly replaced. The resonator is located in the flow path between the fill capillary and the film reservoir so trapping of a persistent current during this process is not surprising. Although the mere presence of a persistent current is a matter of chance since we have no controllable way to produce it, history-dependent splitting occurring at the same third-sound velocity rule out purely geometrical asymmetries. Geometrical asymmetries will scale with the wave number $k=x_{nm}/a$ which depends only on the size of the resonator and of course does not change. It is also interesting to note here that the larger speed at the perimeter [Fig. 1(b)] gives, assuming the $1/r$ flow field, 265 cm/s at the edge of the small hole in the center of the resonator. This is slightly lower than the expected critical velocity $\hbar/2m_4 d$ at this film thickness d (about 311 cm/s) (Ref. 14) reflecting the violent nature of the sample filling process. This was also the largest flow velocity observed.

These splittings, however, are not completely due to the persistent current. There is, and in fact must always be, a contribution from geometrical asymmetries. No real system is ever perfectly degenerate. It is this realization that sheds light on previous failures to see a splitting, so it is worthwhile to consider the interplay between both types of splittings. The analysis is the same as any simple two-level system since the coupling between modes other than the split $\pm m$ modes is negligible. Considering the frequency as the eigenvalues of a 2×2 matrix, the frequency shift given by (2) will be diagonal elements in the rotating basis but a geometrical splitting must be diagonal in a standing wave basis since it is a consequence of the shape of the fixed resonator. This splitting will appear as off-diagonal elements in the rotating basis. Assuming for the time being that the geometrical asymmetry is due to a small deviation from a circular perimeter, i.e., that the radius of the resonator edge is given by $r(\phi)=a[1+\eta(\phi)]$ with $\eta(\phi)\ll 1$, first-order perturbation of boundary conditions in the degenerate case gives

$$\Delta=\pm|\Delta_m|, \quad \Delta_m=\frac{1}{2\pi}\int_0^{2\pi}\eta(\phi)e^{-2im\phi}d\phi, \tag{3}$$

for the relative shift of the modes. The resonant frequencies and normal modes of a resonator with both a circulation splitting and a geometrical asymmetry are, therefore, given by the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} \omega_0+\gamma V_0 k & \omega_0\Delta_m^* \\ \omega_0\Delta_m & \omega_0-\gamma V_0 k \end{pmatrix}. \tag{4}$$

The resonant frequencies are easily found as

$$\omega=\omega_0\pm[(\gamma V_0 k)^2+(\omega_0\Delta)^2]^{1/2}. \tag{5}$$

The resonant frequencies versus V_0 are plotted in Fig. 2. At low circulation speeds the splitting is determined by the geometrical asymmetry and the eigenmodes are standing waves. At high circulation speeds the splitting is as in

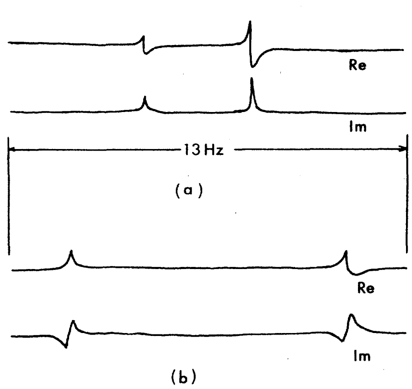

FIG. 1. Third-sound resonances showing the circulation induced splitting of the $n=1$, $m=1$ mode. The sound velocity in both cases is 24 m/s yet the splitting is different reflecting different flow states. The resonance would remain split even with no flow because of geometrical asymmetries.

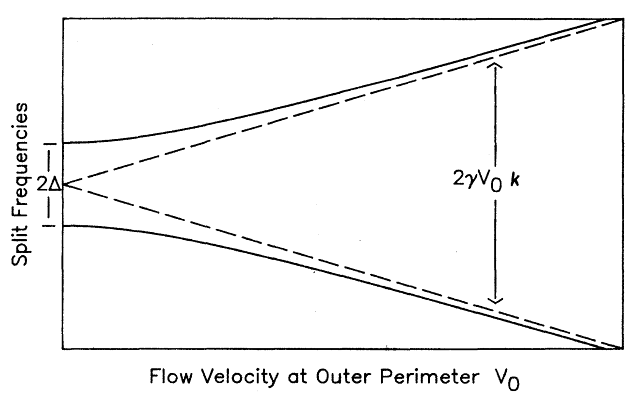

FIG. 2. The solid lines show the normalized resonant frequencies vs the size of the circulation calculated from Eq. (5). The dashed lines represent only the Doppler shift. Small circulations would be obscured by the asymmetry shift Δ. For the $n=1$, $m=1$ modes, $\gamma=1.277\ldots$ The frequency scale is offset to emphasize the splittings.

(1) and the eigenmodes are the traveling (rotating) waves. It is also important to note here that the geometrical splitting is a fixed ratio with respect to the resonant frequency whereas the Doppler shift is a fixed difference. Because of this, (4) provides a clear separation of the two effects via a plot of $(\Delta f)^2$ vs f^2, assuming V_0 remains constant during gradual changes in thickness (hence resonant frequency). Trapping of flow velocity during thickness changes in this manner has been verified in other studies of persistent currents,[15,16] and allows us to investigate the presence of a circulation without a controllable method for changing the flow state. Such a plot is shown as Fig. 3. The perimeter speed in this case is found to be $V_0=1.94$ cm/s corresponding to approximately 7150 circulation quanta. The geometrical asymmetry splitting is $\Delta=9.4\times10^{-4}$. If, for example, the resonator were elliptical, the radius would deviate from a 1.17-cm diam circle by only 11 μm. Referring again to Fig. 1, based on the above measure of Δ, the splittings are 35% rotational in (a) and 74% in (b). This also accounts for the smaller splitting [Fig. 1(a)] showing a more noticeable difference between the amplitudes of the split pair. The standing-wave modes overlap the transducer differently according to the orientation of the asymmetry whereas the rotating states couple to the transducer equally.

We can now propose one possible explanation for the lack of a splitting in other resonator experiments. If there is no degeneracy at $V_0=0$, the circulation splitting is then second order in V_0 and would be visible only at larger velocities. If Telschow, Rudnick, and Wang,[3] for example, had their expected splitting of about 10^{-4} washed out by an asymmetry, Δ would have to be on the order of 10^{-4} or larger. This is quite possible, considering that this is still nine times smaller than the resonator in this work. The question is then: If the asymmetry is dominating, where is the other mode? Considering the typical size of the shift (3), it is somewhat surprising that geometrical splitting of a circular resonator has not been previously reported, especially given the high Q's attainable in these resonators. We believe the transducers themselves may be influential enough to force the modes into the standing-wave configuration, either by directly perturbing the uniformity of the substrate by their mere presence or indirectly through heating, as would be the case with thermal drives and pickups. If so, the modes would naturally split into one strongly coupled to the transducers and the other weakly coupled and hence not visible. This would be even more likely in a resonator where the transducers drastically differ from the surrounding surface as opposed to the integral transducers used here where only thin gaps separate electrodes from identical neutral surfaces.

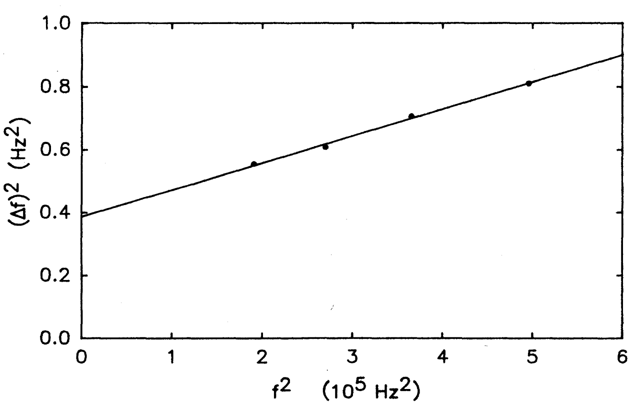

FIG. 3. The observed frequency shifts plotted vs resonant frequency as the film was thickened (frequency lowered) with a fixed flow state. Squaring the frequency scales gives a straight line plot expected from Eq. (5). For these data, the intercept gives a perimeter flow speed $V_0=1.94$ cm/s and the slope gives the asymmetry shift $\Delta=9.4\times10^{-4}$. The size of the filled circles represent the uncertainty of the data.

One other explanation may be related to the topology of the resonator surface. In a resonator with only one hole or no hole at all, the behavior of the velocity field near the $1/r$ singularity will undoubtedly disperse the flow field, since singly quantized vorticies are the preferred state. Uncertainties in this regard are avoided in a multiply connected resonator used where the singularity is avoided completely.

It is evident from Fig. 2 that geometrical asymmetries will put a lower limit on the size of circulation splittings which can be seen with a resonator provided there is no other limiting factor such as the width of the resonance. With even generous assumptions of $C_3=1$ m/s and $a=1$ mm, this restricts Δ to less than 2×10^{-5} if splittings on the order of single-circulation quantum are to be observed. This would be difficult to attain but not impossible. Another consideration, as mentioned above, is the resonance width. Even if the asymmetry splitting were absent, the resonance widths shown in Fig. 1 are roughly a factor of 100 too large. This width, related to the dissipation mechanisms in the film, depends on the drive amplitude and has been observed to be 40 times smaller at the lowest possible drive corresponding to a Q of 400000. The scans shown in Fig. 1 were taken at a high drive so that the signal was large enough to permit both resonances to appear on the same scan in a reasonable time. The nonlinear behavior and unusually large quality factor will be the subject of a subsequent paper. We do not yet know whether a smaller diameter resonator needed to enhance the circulation splitting of a single quantum to a more reasonable size would have an equally large Q.

In conclusion, we have observed a circulation-induced splitting in a third-sound resonator and discussed its interaction with geometrically induced splittings. Equation (5) provides a simple method of determining each contribution. Neither effect has been previously reported for a third-sound mode. The circulation splitting cannot be seen if it is substantially smaller than the geometrical splitting since it becomes second order in the film flow velocity. Care must be taken to keep the transducers nonintrusive and hence sensitive to all modes of interest. Third-sound resonators may now provide more accurate measurements of persistent currents via the Doppler shift provided proper consideration of resonator asymmetries is taken. The observation of quantum steps in the circulation should be possible in an appropriately designed resonator.

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